18. Datuak: $v_0 = 15$ m/s

a) Irismenari dagokion formula erabiliko dugu.

$$x = \frac{v_0^2}{g} \sin 2\alpha$$

30°-ko angeluaz:

$$x = \frac{\left(15\ \frac{m}{s}\right)^2}{9{,}8\ \frac{m}{s^2}} \sin\ (2 \cdot 30) = 19{,}9\ m$$

45°-ko angeluaz:

$$x = \frac{\left(15\ \frac{m}{s}\right)^2}{9{,}8\ \frac{m}{s^2}} \sin\ (2 \cdot 45) = 23\ m$$

60°-ko angeluaz:

$$x = \frac{\left(15\ \frac{m}{s}\right)^2}{9{,}8\ \frac{m}{s^2}} \sin\ (2 \cdot 60) = 19{,}9\ m$$

b) $x = x_0 + v_{0\,x}(t - t_0)$ formulatik abiatuz, baloiak airean irauten duen denbora kalkula dezakegu. $x_0 = 0$ m eta $t_0 = 0$ s direnez:

$$x = v_{0\,x}\, t\ ;\ t = \frac{x}{v_{0\,x}} = \frac{x}{v \cos \alpha}$$

30°-ko angeluaz:

$$t = \frac{19{,}9\ \cancel{m}}{15\ \frac{\cancel{m}}{s} \cos 30} = 1{,}5\ s$$

45°-ko angeluaz:

$$t = \frac{23\ \cancel{m}}{15\ \frac{\cancel{m}}{s} \cos 45} = 2{,}2\ s$$

60°-ko angeluaz:

$$t = \frac{19{,}9\ \cancel{m}}{15\ \frac{\cancel{m}}{s} \cos 60} = 2{,}7\ s$$

HIGIDURA ZIRKULARRA

19. • Egia, puntu guztiak zentrotik distantzia berberera daudelako; beraz, abiadura, $v = \omega R$, berbera da guztien kasuan.

 • Gezurra. Abiadura lineala gurpilaren zentroraino dagoen distantziaren mendekoa da; beraz, erradioko puntu bakoitzak abiadura lineal desberdina du.

 • Egia. Abiaduraren norabidea aldatuz doanez, azelerazio normala dago.

20. Puntu guztiek azelerazio angeluar berbera dute, angelu berdineko biraketak egiten dituztelako denbora-tarte berdinetan; baina ez dute azelerazio tangentzial berdinik, azelerazio hori erradioaren mendekoa delako.

21. Datuak: $\omega = 90$ bira/min ; $R = 0{,}75$ m

a) $$90\ \text{bira/min} = \frac{90\ \cancel{\text{bira}}}{1\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ s} \cdot \frac{2\pi\ \text{rad}}{1\ \cancel{\text{bira}}} = 3\pi\ \text{rad/s}$$

b) $$v = \omega\, R = 3\ \pi\ \frac{\text{rad}}{s} \cdot 0{,}75\ m = 7{,}1\ m/s$$

22. Datuak: $R = 0{,}15$ m ; $w = 33$ bira/min

a) $$33\ \text{bira/min} = \frac{33\ \cancel{\text{bira}}}{1\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ s} \cdot \frac{2\pi\ \text{rad}}{1\ \cancel{\text{bira}}} = 1{,}1\pi\ \text{rad/s}$$

b) $$v = \omega\, R = 1{,}1\ \pi\ \frac{\text{rad}}{s} \cdot 0{,}15\ m = 0{,}5\ m/s$$

c) Higidura zirkular uniformearen ekuazioa erabiliz, eta 5 min = 300 s dela kontuan hartuz:

$$\varphi = \omega\, t = 1{,}1\pi\ \frac{\text{rad}}{\cancel{s}} \cdot 300\ \cancel{s} = 330\ \pi\,\text{rad}$$

$$330\ \pi\,\text{rad} = 330\,\cancel{\pi}\,\cancel{\text{rad}}\ \frac{1\ \text{birabete}}{2\,\cancel{\pi}\,\cancel{\text{rad}}} = 165\ \text{bira}$$

23. Datuak: $\omega = 42$ b/min ; $R = 0{,}4$ m

a) $$42\ \text{bira/min} = \frac{42\ \cancel{\text{bira}}}{1\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ s} \cdot \frac{2\pi\ \text{rad}}{1\ \cancel{\text{bira}}} = $$
$$= 1{,}4\ \pi\,\text{rad/s}$$

b) $$a_n = \omega^2 R = \left(1{,}\ 4\ \pi\ \frac{\text{rad}}{s}\right)^2 0{,}4\ m = 7{,}7\ m/s^2$$

c) 4 min = 240 s

$$\varphi = \omega t = 1{,}4\ \pi\ \frac{\text{rad}}{\cancel{s}} \cdot 240\ \cancel{s} = 336\ \pi\ \text{rad}$$

$$336\ \cancel{\pi}\ \cancel{\text{rad}} \cdot \frac{1\ \text{birabete}}{2\,\cancel{\pi}\,\cancel{\text{rad}}} = 168\ \text{bira}$$

24. Datuak: $\Delta s = 10\,260$ m ; $\Delta t = 45$ min $= 2\,700$ s

$D = 0{,}8$ m

Gurpilen erradioa: $R = \frac{D}{2} = 0{,}4$ m

a) $$\Delta s = R\,\Delta\varphi\ ;\ \Delta\varphi = \frac{\Delta s}{R}$$

$$\Delta\varphi = \frac{10\,260\ \cancel{m}}{0{,}4\ \cancel{m}} = 25\,650\ \text{rad}$$

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{25\,650\ \text{rad}}{2\,700\ s} = 9{,}5\ \text{rad/s}$$

b) $\Delta\varphi = 25\,650$ rad, *a* atalean azaldu denez.

25. Datuak: $\omega_0 = 300$ bira/min ; $t = 10$ s ; $\omega = 0$ bira/min ; $t_0 = 0$ s

a) $$\omega_0 = \frac{300 \text{ bira}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{2\pi \text{ rad}}{1 \text{ bira}} = 10\ \pi \text{ rad/s}$$

$$\alpha = \frac{\omega - \omega_0}{t - t_0} = \frac{0 - 10\ \pi \text{ rad} \cdot \text{s}^{-1}}{10 \text{ s} - 0 \text{ s}} = -\pi \text{ rad/s}^2$$

b) $\omega = \omega_0 + \alpha\,(t - t_0)$

$$\omega = 10\ \pi \frac{\text{rad}}{\text{s}} + \left(-\pi \frac{\text{rad}}{\text{s}^2}\right) 4 \text{ s} = 6\ \pi \text{ rad/s}$$

c) Higidura zirkular uniformeki azeleratuaren ekuazioa aplikatuz:

$$\varphi = \varphi_0 + \omega_0\,(t - t_0) + \frac{1}{2}\,\alpha\,(t - t_0)^2$$

$$\varphi = 10\pi \frac{\text{rad}}{\text{s}} \cdot 10 \text{ s} - \frac{1}{2}\,\pi \frac{\text{rad}}{\text{s}^2} \cdot (10 \text{ s})^2 = 50\pi \text{ rad}$$

$$50\pi \text{ rad}\,\frac{1 \text{ bira}}{2\pi \text{ rad}} = 25 \text{ bira}$$

26. Datuak: $\omega_0 = 0$ rad/s ; $\omega = 5$ rad/s ; $t_0 = 0$ s
$t = 1$ min $= 60$ s ; $R = 0{,}15$ m

a) $$\alpha = \frac{\omega - \omega_0}{t - t_0} = \frac{5 \frac{\text{rad}}{\text{s}}}{60 \text{ s}} = 0{,}08 \text{ rad/s}^2$$

b) Abiadura lineala kalkulatzeko, higidura hasi eta 25 s pasatu ondoren izango den abiadura angeluarra kalkulatuko dugu lehenik:

$$\omega = \omega_0 + \alpha\,(t - t_0)$$

$$\omega = 0{,}08 \frac{\text{rad}}{\text{s}^2} \cdot 25 \text{ s} = 2 \text{ rad/s}$$

$$v = \omega\,R = 2 \frac{\text{rad}}{\text{s}} \cdot 0{,}15 \text{ m} = 0{,}3 \text{ m/s}$$

c) $$a_t = \alpha\,R = 0{,}08 \frac{\text{rad}}{\text{s}^2}\,0{,}15 \text{ m} = 0{,}01 \text{ m/s}^2$$

d) $$\varphi = \varphi_0 + \omega_0\,(t - t_0) + \frac{1}{2}\,\alpha\,(t - t_0)^2$$

$$\varphi = \frac{1}{2}\,\alpha\,t^2 = \frac{1}{2}\,0{,}08 \frac{\text{rad}}{\text{s}^2}\,(60 \text{ s})^2 = 144 \text{ rad}$$

$$144 \text{ rad} = 144 \text{ rad}\,\frac{1 \text{ bira}}{2\pi \text{ rad}} = 22{,}9 \text{ bira}$$

JARDUERA ETA PROBLEMA EBATZIAK

1. *a)* 1., 3. eta 5. tarteetan abiadura uniformeki aldatu da denbora pasatu ahala; beraz, higidura HZUA motakoa izan da.

2. eta 4. tarteetan abiadura ez da aldatu; beraz, HZUa izan da.

b) 1. tartean:

$$a = \frac{v - v_0}{t - t_0} = \frac{10 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{5 \text{ s} - 0 \text{ s}} = 2 \text{ m/s}^2$$

2. tartean:

$a = 0$ m/s², HZUa delako.

3. tartean:

$$a = \frac{20 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}}}{20 \text{ s} - 10 \text{ s}} = 1 \text{ m/s}^2$$

4. tartean:

$a = 0$ m/s², HZUa delako.

5. tartean:

$$a = \frac{0 \frac{\text{m}}{\text{s}} - 20 \frac{\text{m}}{\text{s}}}{35 \text{ s} - 25 \text{ s}} = -2 \text{ m/s}^2$$

Higikaria balaztatzen ari delako jarri dugu minus zeinua.

c) 1. tartea

$$x_1 = x_0 + v_0\,(t - t_0) + \frac{1}{2}\,a\,(t - t_0)^2$$

baldin $x_1 - x_0 = \Delta x_1$ bada, orduan:

$$\Delta x_1 = \frac{1}{2} \cdot 2 \frac{\text{m}}{\text{s}^2}\,(5 \text{ s} - 0 \text{ s})^2 = 25 \text{ m}$$

2. tartea

$$\Delta x_2 = v_0\,(t - t_0) = 10 \frac{\text{m}}{\text{s}}\,(10 \text{ s} - 5 \text{ s}) = 50 \text{ m}$$

3. tartea

$$\Delta x_3 = v_0\,(t - t_0) + \frac{1}{2}\,a\,(t - t_0)^2$$

$$\Delta x_3 = 10 \frac{\text{m}}{\text{s}}\,(20 \text{ s} - 10 \text{ s}) + \frac{1}{2}\,1 \frac{\text{m}}{\text{s}^2}\,(20 \text{ s} - 10 \text{ s})^2 = 150 \text{ m}$$

4. tartea:

$$\Delta x_4 = v_0\,(t - t_0) = 20 \frac{\text{m}}{\text{s}}\,(25 \text{ s} - 20 \text{ s}) = 100 \text{ m}$$

5. tartea:

$$\Delta x_5 = v_0\,(t - t_0) + \frac{1}{2}\,a\,(t - t_0)^2$$

$$\Delta x_5 = 20 \frac{\text{m}}{\text{s}}\,(35 \text{ s} - 25 \text{ s}) + \frac{1}{2}\left(-2 \text{ s} \frac{\text{m}}{\text{s}^2}\right)(35 \text{ s} - 25 \text{ s})^2 = 100 \text{ m}$$