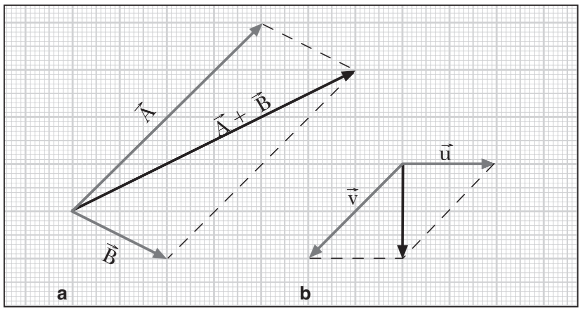
# 4 Indarrak

**ATARIAN**

- 


- Pitagoras-en teorema aplikatuz:

  *a)* $c^2 = a^2 + b^2 = (5\ \text{m})^2 + (9\ \text{m})^2 = 106\ \text{m}^2$

  $c = \sqrt{106\ \text{m}^2} = 10{,}3\ \text{m}$

  *b)* $c^2 = a^2 + b^2$

  $b^2 = c^2 - a^2$

  $b^2 = (20\ \text{cm})^2 - (12\ \text{cm})^2 = 256\ \text{cm}^2$

  $b = \sqrt{256\ \text{cm}^2} = 16\ \text{cm}$

**INDARREN IZAERA**

1. *a)* $19{,}6\ \text{N} = 19{,}6\ \cancel{\text{N}}\ \dfrac{1\ \text{kp}}{9{,}8\ \cancel{\text{N}}} = 2\ \text{kp}$

   *b)* $28\ \text{kp} = 28\ \cancel{\text{kp}}\ \dfrac{9{,}8\ \text{N}}{1\ \cancel{\text{kp}}} = 274{,}4\ \text{N}$

2. Jonek:

$$23\ \text{kp} = 23\ \cancel{\text{kp}}\ \frac{9{,}8\ \text{N}}{1\ \cancel{\text{kp}}} = 225{,}4\ \text{N}$$

   Maitek:

$$230\ \text{N} > 225{,}4\ \text{N}$$

   Maitek indar handiagoa egiten du.

3. Datuak:

   $K = 100\ \dfrac{\text{N}}{\text{m}}$ ; $F = 85\ \text{N}$

   $F = K \Delta l$

$$\Delta l = \frac{F}{K} = \frac{85\ \cancel{\text{N}}}{100\ \dfrac{\cancel{\text{N}}}{\text{m}}} = 0{,}85\ \text{m} = 85\ \text{cm}$$

4. Datuak:

   $K = 150\ \dfrac{\text{N}}{\text{m}}$ ; $l_0 = 35\ \text{cm} = 0{,}35\ \text{m}$

   *a)* $l = 0{,}45$ m bada,

$$F = K(l - l_0) = 150\ \frac{\text{N}}{\cancel{\text{m}}}\ (0{,}45\ \cancel{\text{m}} - 0{,}35\ \cancel{\text{m}}) = 15\ \text{N}$$

   *b)* $F = 63$ N bada,

   $F = K(l - l_0)$

$$l = \frac{F}{K} + l_0 = \frac{63\ \cancel{\text{N}}}{150\ \dfrac{\cancel{\text{N}}}{\text{m}}} + 0{,}35\ \text{m} = 0{,}77\ \text{m} = 77\ \text{cm}$$

5. Datuak: $l - l_0 = 12\ \text{cm} = 0{,}12\ \text{m}$ ; $F = 18\ \text{N}$

   *a)* $F = K \Delta l$

$$K = \frac{F}{\Delta l} = \frac{18\ \text{N}}{0{,}12\ \text{m}} = 150\ \frac{\text{N}}{\text{m}}$$

   *b)* $F = K \Delta l$

$$\Delta l = \frac{F}{K} = \frac{45\ \cancel{\text{N}}}{150\ \dfrac{\cancel{\text{N}}}{\text{m}}} = 0{,}3\ \text{m} = 30\ \text{cm}$$

6. Datuak:

   $K = 240\ \dfrac{\text{N}}{\text{m}}$ ; $l = 35\ \text{cm} = 0{,}35\ \text{m}$ ; $F = 48\ \text{N}$

   Malgukiari Hooke-ren legea aplikatuz, $l_0$ lortuko dugu:

$$F = K(l - l_0)$$

$$l_0 = -\frac{F}{K} + l = -\frac{48\ \cancel{\text{N}}}{240\ \dfrac{\cancel{\text{N}}}{\text{m}}} + 0{,}35\ \text{m} = 0{,}15\ \text{m}$$

   Hooke-ren legea aplikatuz indarra ere lortuko dugu:

$$F = K(l - l_0) = 240\ \frac{\text{N}}{\text{m}}\ (0{,}4\ \text{m} - 0{,}15\ \text{m}) = 60\ \text{N}$$

7. Datuak: $l_0 = 25 \text{ cm} = 0{,}25 \text{ m}$ ; $l = 45 \text{ cm} = 0{,}45 \text{ m}$
$F = 22 \text{ N}$

*a)* $F = K(l - l_0)$

$$K = \frac{F}{l - l_0} = \frac{22 \text{ N}}{0{,}45 \text{ m} - 0{,}25 \text{ m}} = 110 \frac{\text{N}}{\text{m}}$$

*b)* *l*-ren balioa kalkulatzeko, bakanduko dugu Hooke-ren legean:

$$F = K(l - l_0)$$

$$l = l_0 + \frac{F}{K} = 0{,}25 \text{ m} + \frac{27{,}5 \cancel{\text{N}}}{110 \frac{\cancel{\text{N}}}{\text{m}}} = 0{,}5 \text{ m} = 50 \text{ cm}$$

8. Datuak: $l_1 = 32 \text{ cm} = 0{,}32 \text{ m}$ ; $F_1 = 1{,}2 \text{ N}$
$l_2 = 40 \text{ cm} = 0{,}4 \text{ m}$ ; $F_2 = 1{,}8 \text{ N}$

Hooke-ren legea aplikatuko diegu bi indarrei:

$$F_1 = K(l_1 - l_0)$$
$$F_2 = K(l_2 - l_0)$$

Enuntziatuko datuak ordeztuz:

$$1{,}2 \text{ N} = K(0{,}32 \text{ m} - l_0)$$
$$1{,}8 \text{ N} = K(0{,}4 \text{ m} - l_0)$$

*a)* Ekuazio hauetan *K* bakanduz eta bi adierazpenak berdinduz:

$$K = \frac{1{,}2 \cancel{\text{N}}}{0{,}32 \text{ m} - l_0} = \frac{1{,}8 \cancel{\text{N}}}{0{,}4 \text{ m} - l_0}$$

$$1{,}2\ (0{,}40 \text{ m} - l_0) = 1{,}8\ (0{,}32 \text{ m} - l_0)$$

$$1{,}2 \cdot 0{,}40 \text{ m} - 1{,}2\ l_0 = 1{,}8 \cdot 0{,}32 \text{ m} - 1{,}8\ l_0$$

$l_0$ bakanduz:

$$l_0 = \frac{1{,}2 \cdot 0{,}4 \text{ m} - 1{,}8 \cdot 0{,}32 \text{ m}}{1{,}2 - 1{,}8} = 0{,}16 \text{ m} = 16 \text{ cm}$$

*b)* Konstante elastikoa Hooke-ren legearen bidez kalkulatuko dugu:

$$F = K(l - l_0)$$

$$K = \frac{F_1}{l_1 - l_0} = \frac{1{,}2 \text{ N}}{0{,}32 \text{ m} - 0{,}16 \text{ m}} = 7{,}5 \frac{\text{N}}{\text{m}}$$

**INDAR-SISTEMA BATEN INDAR ERRESULTANTEA**

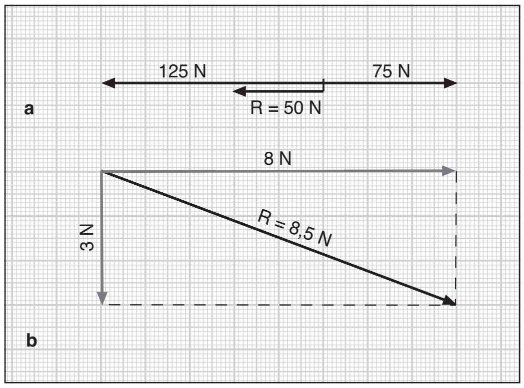
9. Indarrek modulu bera baina aurkako noranzkoa izan behar dituzte.

10. *a)* Datuak: $F_1 = 125 \text{ N}$ ; $F_2 = 75 \text{ N}$

$$R = F_1 - F_2 = 125 \text{ N} - 75 \text{ N} = 50 \text{ N}$$

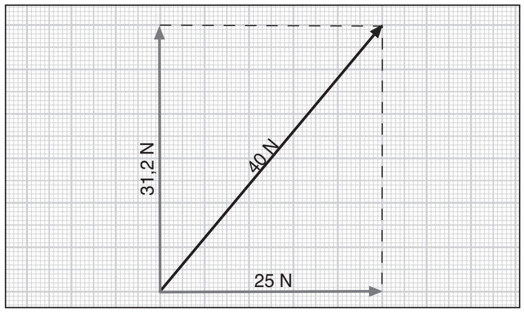
*b)* Datuak: $F_1 = 8 \text{ N}$ ; $F_2 = 3 \text{ N}$

$$R = \sqrt{(8 \text{ N})^2 + (3 \text{ N})^2} = 8{,}5 \text{ N}$$




11. Datuak: $F_1 = 25 \text{ N}$ ; $R = 40 \text{ N}$

*a)*



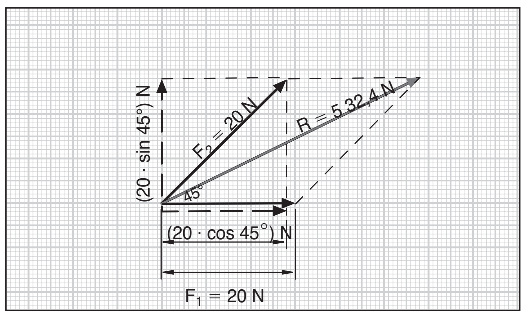

*b)* Pitagoras-en teorematik $F_2$ bakanduz:

$$R^2 = F_1^2 + F_2^2$$

$$F_2 = \sqrt{R^2 - F_1^2} = \sqrt{(40 \text{ N})^2 - (25 \text{ N})^2} = 31{,}2 \text{ N}$$

12. Datuak: $F_1 = 15 \text{ N}$ ; $F_2 = 20 \text{ N}$ ; $\alpha = 45°$

*a)*




*b)* Bektoreen osagaiak irudian azaltzen den bezala kalkulatuko ditugu:

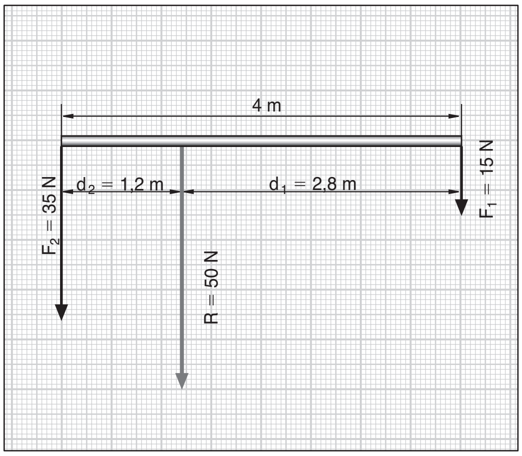
$$F_{1x} = 15$$
$$F_{1y} = 0 \text{ N}$$
$$F_{2x} = F_2 \cdot \cos \alpha = (20 \cos 45°) \text{ N} = 14{,}14 \text{ N}$$
$$F_{2y} = F_2 \cdot \cos \alpha = (20 \sin 45°) \text{ N} = 14{,}14 \text{ N}$$
$$R_x = (15 + 14{,}14) \text{ N} = 29{,}14 \text{ N}$$
$$R_y = 14{,}14 \text{ N}$$
$$\vec{R} = (29{,}14\ \vec{i} + 14{,}14\ \vec{j}) \text{ N}$$

$\vec{R}$ bektorearen modulua kalkulatuko dugu:

$$R = \sqrt{29{,}14^2 + 14{,}14^2}\ \text{N} = 32{,}4\ \text{N}$$

13. Datuak: $d_1 + d_2 = 4$ m ; $F_1 = 15$ N ; $F_2 = 35$ N

*a)*




*b)* Indar erresultanteak modulu hau du:

$$R = F_1 + F_2 = (15 + 35)\ \text{N} = 50\ \text{N}$$

Aplikazio-puntua bilatuko dugu:

$$d_1 + d_2 = 4\ \text{m}\ ;\ d_1 = 4\ \text{m} - d_2$$

Bi ekuazio eta bi ezezagun ditugu:

$$\begin{cases} d_1 = 4\ \text{m} - d_2 \\ F_1\, d_1 = F_2\, d_2 \end{cases}$$

$$F_1\,(4\ \text{m} - d_2) = F_2\, d_2$$

$$F_1 \cdot 4\ \text{m} = (F_2 + F_1)\, d_2$$

$$d_2 = \frac{F_1 \cdot 4\ \text{m}}{F_2 + F_1} = \frac{15\ \cancel{\text{N}} \cdot 4\ \text{m}}{35\ \cancel{\text{N}} + 15\ \cancel{\text{N}}} = 1{,}2\ \text{m}$$

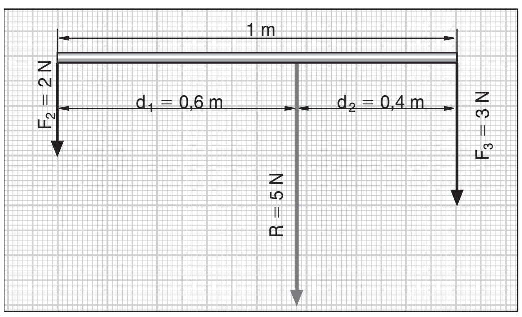
Azkenik, $d_1$ bakanduko dugu adierazpen honetan:

$$d_1 + d_2 = 4\ \text{m}$$

$$d_1 = 4\ \text{m} - d_2 = 4\ \text{m} - 1{,}2\ \text{m} = 2{,}8\ \text{m}$$

14. Datuak: $F_1 = 2$ N ; $F_2 = 3$ N ; $d_1 + d_2 = 1$ m

*a)*




*b)* Indar erresultantearen modulua kalkulatuko dugu:

$$R = F_1 + F_2 = 2\ \text{N} + 3\ \text{N} = 5\ \text{N}$$

Bi ezezaguneko bi ekuazioren sistema bat dugu, aplikazio-puntua lortzeko balioko diguna:

$$\begin{cases} d_1 + d_2 = 1\ \text{m} \rightarrow d_1 = 1\ \text{m} - d_2 \\ F_1\, d_1 = F_2\, d_2 \end{cases}$$

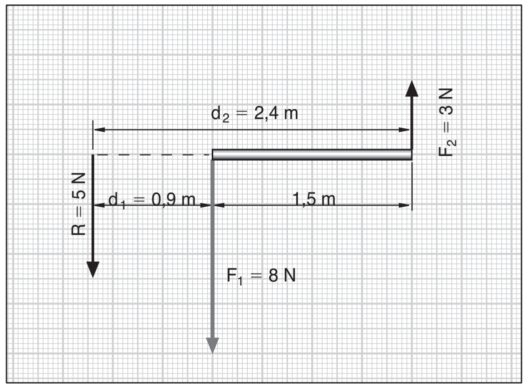
$$F_1\,(1\ \text{m} - d_2) = F_2\, d_2$$

$$d_2 = \frac{1\ \text{m} \cdot F_1}{F_1 + F_2} = \frac{1\ \text{m} \cdot 2\ \text{N}}{2\ \text{N} + 3\ \text{N}} = 0{,}4\ \text{m}$$

$$d_1 = 1\ \text{m} - 0{,}4\ \text{m} = 0{,}6\ \text{m}$$

15. Datuak: $d_2 - d_1 = 1{,}5$ m ; $F_1 = 8$ N ; $F_2 = 3$ N

*a)*




*b)* Indar erresultantearen modulua honako hau da:

$$R = F_1 - F_2 = 8\ \text{N} - 3\ \text{N} = 5\ \text{N}$$

Aplikazio-puntua lortzeko, $d_1$ eta $d_2$ bakanduko ditugu ekuazio-sistema honetan:

$$\begin{cases} d_2 - d_1 = 1{,}5\ \text{m} \rightarrow d_2 = 1{,}5\ \text{m} + d_1 \\ F_1\, d_1 = F_2\, d_2 \end{cases}$$

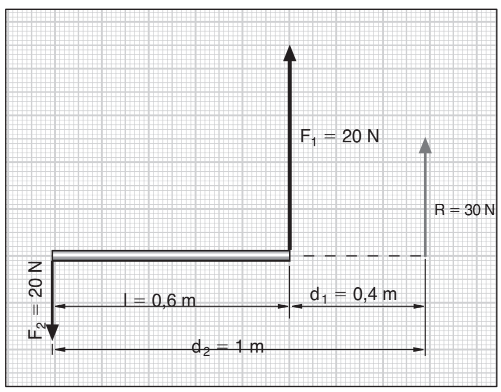
$$F_1\, d_1 = F_2\,(1{,}5\ \text{m} + d_1)$$

$$d_1 = \frac{F_2 \cdot 1{,}5\ \text{m}}{F_1 - F_2} = \frac{3\ \text{N} \cdot 1{,}5\ \text{m}}{8\ \text{N} - 3\ \text{N}} = 0{,}9\ \text{m}$$

$$d_2 = 1{,}5\ \text{m} + 0{,}9\ \text{m} = 2{,}4\ \text{m}$$

16. Datuak: $F_1 = 50$ N ; $F_2 = 20$ N ; $d_2 - d_1 = 0{,}6$ m

*a)*




Aplikazio-puntua lortzeko, ekuazio-sistema hau ebatziko dugu:

$$\begin{cases} d_2 - d_1 = 0{,}6\ \text{m} \rightarrow d_2 = 0{,}6\ \text{m} + d_1 \\ F_1\, d_1 = F_2\, d_2 \end{cases}$$

$$F_1 d_1 = F_2 (0{,}6 \text{ m} + d_1)$$

$$d_1 = \frac{F_2 \cdot 0{,}6 \text{ m}}{F_1 - F_2} = \frac{20 \text{ N} \cdot 0{,}6 \text{ m}}{50 \text{ N} - 20 \text{ N}} = 0{,}4 \text{ m}$$

$$d_2 = 0{,}6 \text{ m} + 0{,}4 \text{ m} = 1 \text{ m}$$

*b)* Indar erresultantearen modulua kalkulatuko dugu:

$$R = 50 \text{ N} - 20 \text{ N} = 30 \text{ N}$$

17. Datuak: $F_1 = 10$ N ; $F_2 = 5$ N

*a)* Bi indarrak elkarren paraleloak eta noranzko berekoak direnez, indar erresultanteak modulua hau du:

$$R = F_1 + F_2 = 10 \text{ N} + 5 \text{ N} = 15 \text{ N}$$

Bi indarrak noranzko berekoak direnez, $d_1 + d_2 = 3$ m, datu horiek erabiliz, ekuazio-sistema hau plantea dezakegu:

$$\begin{cases} d_1 + d_2 = 3 \text{ m} \rightarrow d_1 = 3 \text{ m} - d_2 \\ F_1 d_1 = F_2 d_2 \end{cases}$$

$$F_1 (3 \text{ m} - d_2) = F_2 d_2$$

$$d_2 = \frac{F_1 \cdot 3 \text{ m}}{F_1 + F_2} = \frac{10 \text{ N} \cdot 3 \text{ m}}{10 \text{ N} + 5 \text{ N}} = 2 \text{ m}$$

$$d_1 = 3 \text{ m} - 2 \text{ m} = 1 \text{ m}$$

*b)* Indarrak elkarren paraleloak direnez eta aurkako noranzkoa dutenez, honako hau idatz dezakegu:

$$R = F_1 - F_2 = 10 \text{ N} - 5 \text{ N} = 5 \text{ N}$$

Aplikazio-puntua lortzeko, jakinik indarrek aurkako noranzkoa dutela, ekuazio-sistema hau planteatuko dugu:

$$\begin{cases} d_2 - d_1 = 3 \text{ m} \rightarrow d_2 = 3 \text{ m} + d_1 \\ F_1 d_1 = F_2 d_2 \end{cases}$$

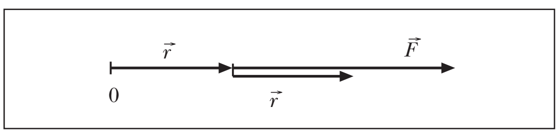
$$F_1 d_1 = F_2 (3 \text{ m} + d_1)$$

$$d_1 = \frac{3 \text{ m} \cdot F_2}{F_1 - F_2} = \frac{3 \text{ m} \cdot 5 \text{ N}}{10 \text{ N} - 5 \text{ N}} = 3 \text{ m}$$

$$d_2 = 3 \text{ m} + d_1 = 3 \text{ m} + 3 \text{ m} = 6 \text{ m}$$

## INDARRAK ETA BIRAKETA-HIGIDURA

18. $\vec{r}$ eta $\vec{F}$ bektoreek eratzen duten angelua nulua da; hots, $\alpha = 0$




Indar-momentua kalkulatuz:

$$M = F r \sin \alpha = F r \sin 0 = F r \cdot 0 = 0$$

Beraz, indar-momentua nulua da.

19. Datuak: $R = 50$ cm $= 0{,}5$ m ; $F = 25$ N ; $\alpha = 90°$

Indar-momentua honako hau da:

$$M = r F \sin \alpha = 0{,}5 \text{ m} \cdot 25 \text{ N} \cdot \sin 90° = \\ = 0{,}5 \cdot \text{m} \cdot 25 \text{ N} \cdot 1 = 12{,}5 \text{ N·m}$$

20. Datuak: $M = 7$ N·m ; $F = 15$ N ; $\alpha = 90°$

$$M = F r \sin \alpha$$

$$r = \frac{M}{F \sin \alpha} = \frac{7 \cancel{\text{N}} \cdot \text{m}}{15 \cancel{\text{N}} \sin 90°} = 0{,}466 \text{ m} = 46{,}6 \text{ cm}$$

21. Datuak: $F_1 = 5$ N ; $r_1 = 10$ m ; $\alpha = 90°$

$F_2 = 12$ N ; $r_2 = 10 \text{ m} - 5 \text{ m} = 5$ m ; $\alpha = 90°$

Lehenik, $M_1$ kalkulatuko dugu:

$$M_1 = F_1 r_1 \sin \alpha = -5 \text{ N} \cdot 10 \text{ m} \cdot \sin 90° = 50 \text{ N·m}$$

non $M_1$ positiboa den, haren ondoriozko biraketak eta erlojuen orratzek egiten dutenak aurkako noranzkoa dutelako.

Ondoren, $M_2$ kalkulatuko dugu:

$$M_2 = -F_2 r_2 \sin \alpha = -12 \text{ N} \cdot 5 \text{ m} \cdot \sin 90° = \\ = -60 \text{ N·m}$$

non $M_2$ negatiboa den, haren ondoriozko biraketak eta erlojuaren orratzek egiten dutenak noranzko bera dutelako.

Azkenik, momentu totala kalkulatuko dugu:

$$M = M_1 + M_2 = 50 \text{ N·m} - 60 \text{ N·m} = -10 \text{ N·m}$$

22. Indar-bikote batek biraketa-higidura eragiten du.

23. Datuak: $F = 20$ N ; $d = 50$ cm $= 0{,}5$ m

$$M = F d = 20 \text{ N} \cdot 0{,}5 \text{ m} = 10 \text{ N·m}$$

Momentuaren modulua: 10 N·m.

24. Datuak: $M = 5$ N·m ; $d = 20$ cm $= 0{,}2$ m

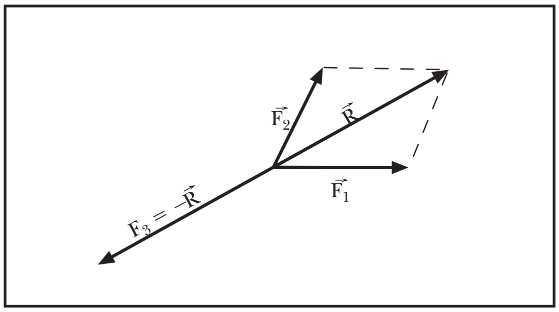
$$M = F d$$

$$F = \frac{M}{d} = \frac{5 \text{ N} \cdot \cancel{\text{m}}}{0{,}2 \cancel{\text{m}}} = 25 \text{ N} ; \quad 25 \text{ N} = 25 \cancel{\text{N}} \frac{1 \text{ kp}}{9{,}8 \cancel{\text{N}}} = 2{,}5 \text{ kp}$$

## OREKAREN BALDINTZA OROKORRAK

25. Bai, biraketa-higidura bat. Adibidez, ziba bat bere ardatzaren inguruan biraka.

26. Diagrama batean bi indar konkurrente marraztuko ditugu, $\vec{F_1}$ eta $\vec{F_2}$, eta ondoren, haien indar erresultantea ere irudikatuko dugu.




Hirugarren indar bat gehituko dugu, oreka-baldintza ezartzeko:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

$$\vec{F_3} = -(\vec{F_1} + \vec{F_2})$$

Beraz, $\vec{F_3}$ indarrak 1. eta 2. indarren erresultantearen modulu eta norabide berberak izan behar ditu, baina aurkako noranzkoa; idazkera matematikoan, honela adieraz daiteke:

$$\vec{F_3} = -\vec{R}$$

27. Hirugarren indar bat gehitu behar dugu, $\vec{F_3}$, gorputzaren biraketa galarazteko. Horretarako, oreka-baldintza hau bete beharko da:

$$\sum_{i=1}^{n} \vec{M_i} = 0$$

$$\vec{M} + \vec{M_3} = 0$$

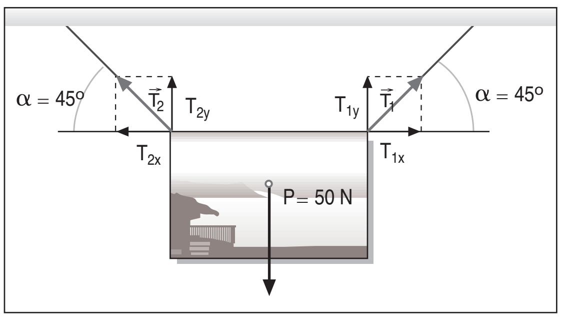
$$\vec{M_3} = -\vec{M}$$

$\vec{M}$ = indar bikotearen momentua

$\vec{M_3}$ = $\vec{F_3}$ indarraren momentua

Beraz, $\vec{F_3}$ indarrak sortuko lukeen biraketa indar bikoteak sortuko lukeenaren aurkakoa da, bi momentuen moduluak berdinak izanik.

28. Datuak:




Oreka estatikorako baldintza aplikatuz:

$$\vec{T_1} + \vec{T_2} + \vec{P} = 0$$

$$T_{1x}\,\vec{i} + T_{1y}\,\vec{j} - T_{2x}\,\vec{i} + T_{2y}\,\vec{j} - P\,\vec{j} = 0$$

Ekuazio bektoriala ardatzen arabera deskonposatuz:

$x$: $T_{1x} - T_{2x} = 0$ ; $T_{1x} = T_{2x}$

$T_1 \cancel{\cos\alpha} = T_2 \cancel{\cos\alpha}$ ; $T_1 = T_2$

$y$: $T_{1y} + T_{2y} - P = 0$

Lehenengo ekuaziotik, bi tentsioen moduluek berdinak izan behar dutela ondorioztatzen da.

Bigarren ekuaziotik tentsioaren modulua ondorioztatzen da:

$$T_{1y} + T_{2y} = P$$

$$T \sin\alpha + T \sin\alpha = P \;;\; 2\,T \sin\alpha = P$$

$$T = \frac{P}{2 \sin\alpha} = \frac{50\ \text{N}}{2 \sin 45°} = 35{,}4\ \text{N}$$

29. Datuak: $T = 110$ N ; $P = 150$ N

Oreka estatikorako baldintzatik:

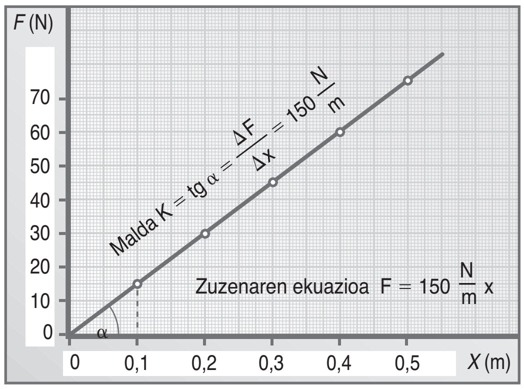
$$2\,T \sin\alpha = P$$

$$\sin\alpha = \frac{P}{2\,T} = \frac{150\ \cancel{\text{N}}}{2 \cdot 110\ \cancel{\text{N}}} = 0{,}68$$

$$\alpha = \arcsin\ (0{,}68) = 43°$$

**JARDUERA ETA PROBLEMA EBATZIAK**

1. *a)*




*b)* Zuzenaren maldak malgukiaren konstante elastikoaren balioa adierazten du:

$$K = \text{tg}\ \alpha = \frac{\Delta F}{\Delta x} = \frac{75\ \text{N} - 15\ \text{N}}{0{,}5\ \text{m} - 0{,}1\ \text{m}} = 150\ \frac{\text{N}}{\text{m}}$$

*c)* Luzera bakanduko dugu Hooke-ren legetik:

$$F = K\,x$$

$$x = \frac{F}{K} = \frac{22{,}5\ \cancel{\text{N}}}{150\ \frac{\cancel{\text{N}}}{\text{m}}} = 0{,}15\ \text{m} = 15\ \text{cm}$$

*d)* Hooke-ren legea aplikatuz:

$$F = K\,x = 150\ \frac{\text{N}}{\cancel{\text{m}}} \cdot 0{,}25\ \cancel{\text{m}} = 37{,}5\ \text{N}$$

2. Aurreko ariketan bezala, zuzenaren malda konstante elastikoa da.

$$K = \text{tg}\ \alpha = \frac{\Delta F}{\Delta x} = \frac{16\ \text{N} - 4\ \text{N}}{4\ \text{m} - 1\ \text{m}} = 4\ \frac{\text{N}}{\text{m}}$$